Single Season Model (Camera-trap Focus)

Part I

USGS
science for a changing world

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Basic Field Situation

- From a population of S sampling units, s are selected and surveyed for the species.
- Units are closed to changes in occupancy during a common 'season'.
- Units must be repeatedly surveyed within a season.
    - Camera-trap surveys: typically temporal replication based on fixed time periods (daily, weekly)

Resulting Data

Unit	1
1	101
2	000
3	100
.	.
.	.
.	.
.	.
s	000

Single Season Model

- ψ = probability a unit is occupied.
- p_j = probability species is detected at a unit in survey j (given presence).

Detection Probability in Camera-trap Studies

- Pr (detect species at unit *i* survey *j*) =

  Pr (unit occupied during season)

  x Pr (at least 1 individual of species passes camera trap during survey *j*)

  x Pr (species detected in survey *j* | unit occupied and at least 1 individual passes camera trap during survey *j*)

Detection Probability in Camera-trap Studies

- Detection probability thus has 2 components:
  - One based on space use
  - One based on camera
    - Camera must be “tripped”
    - Photo must permit species ID

Single Season Model

- Investigating *patterns* in occupancy.

- Variety of approaches all recognizing that an observed ‘absence’ may be the result of a true absence or a nondetection. (e.g., Hewitt 1967, Geissler and Fuller 1987, Azuma et al. 1990, MacKenzie et al. 2002, Tyre et al. 2003, Wintle et al. 2004 and Stauffer et al. 2004)

- MacKenzie et al. (2002) provide most general treatment of the problem.

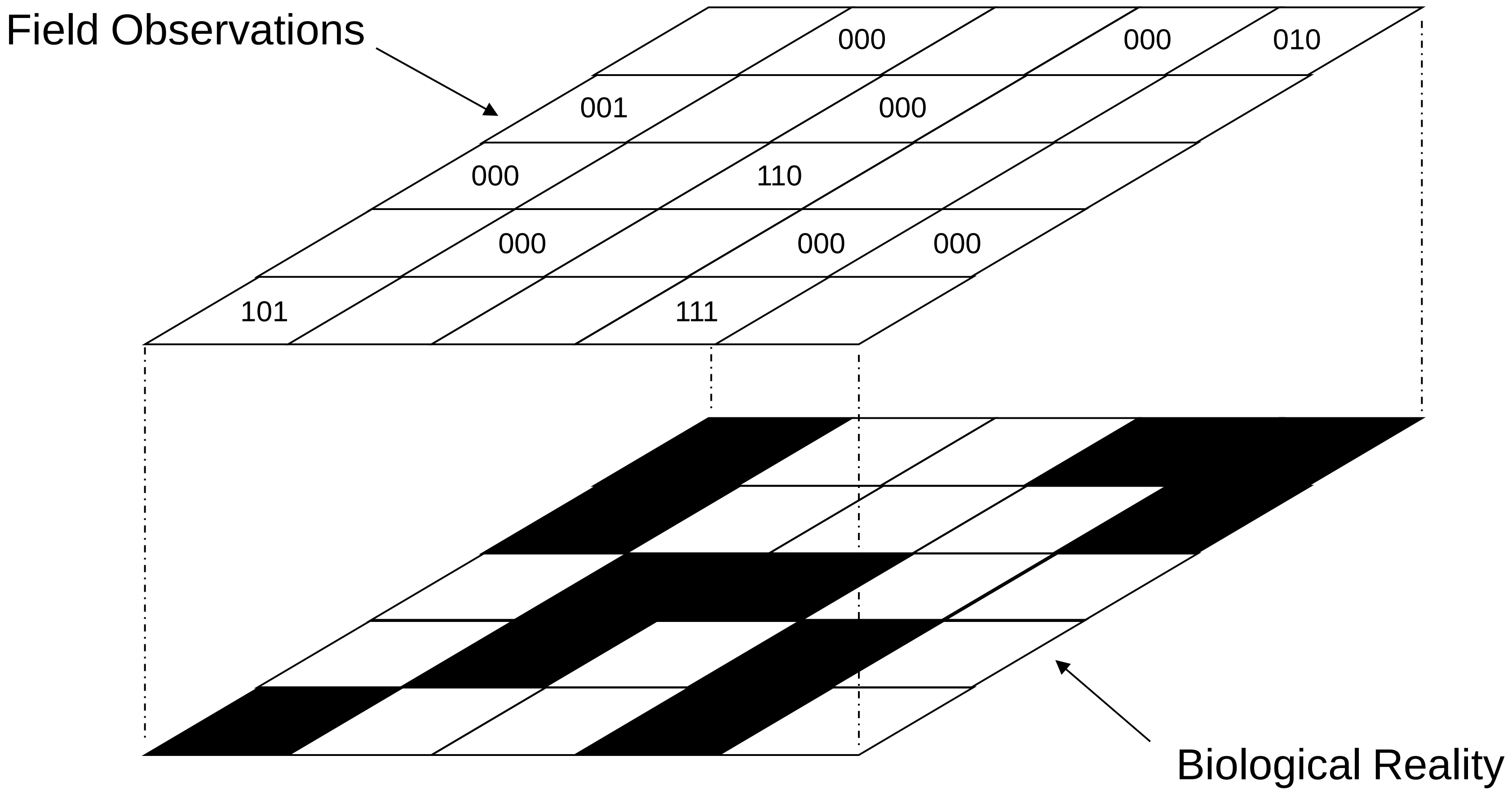
Single Season Model

- Consider the data consists of 2 ‘layers’
  1. True presence/absence of the species.
  2. Observed data which are conditional upon species distribution.

- Knowledge about the first layer is imperfect.

- Must account for the observation process to make reliable inferences about occurrence.

Model Development



Observed Data Likelihood

- Model all possible stochastic processes that may have resulted in observed detection histories.

- Take verbal description of the observed data and translate it into a mathematical equation.

Observed Data Likelihood

For example,

$$\mathbf{h}_1 = 101$$

Verbal description: species is present at the unit, was detected in first and third survey, not detected in second survey.

Mathematical translation:

$$\Pr\left(\mathbf{h}_1 = 101\right) = \psi p_1 \left(1 - p_2\right) p_3$$

Observed Data Likelihood

For example,

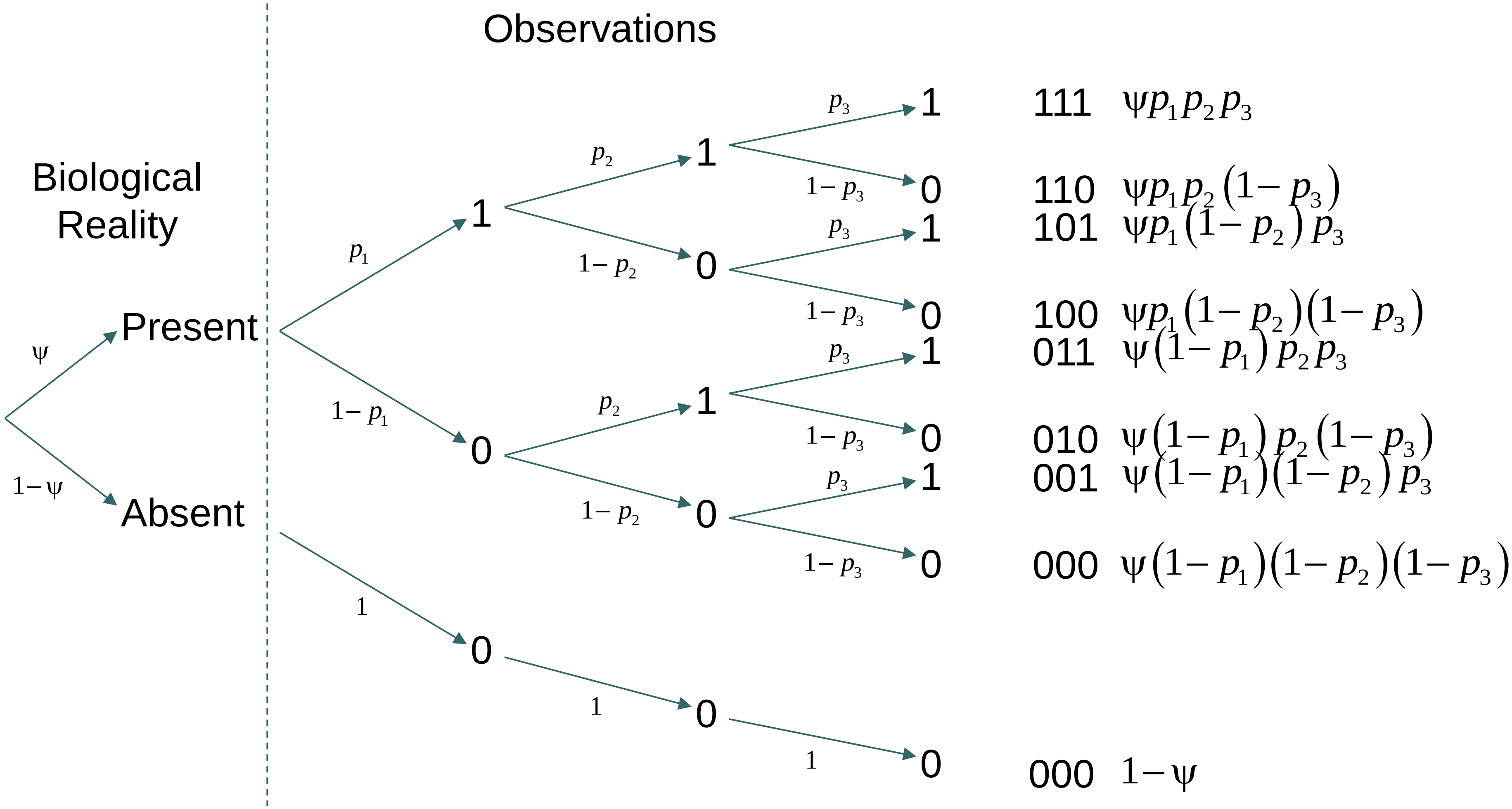
$$\mathbf{h}_2 = 000$$

Verbal description: species is present at the unit and was never detected, *OR* species is absent.

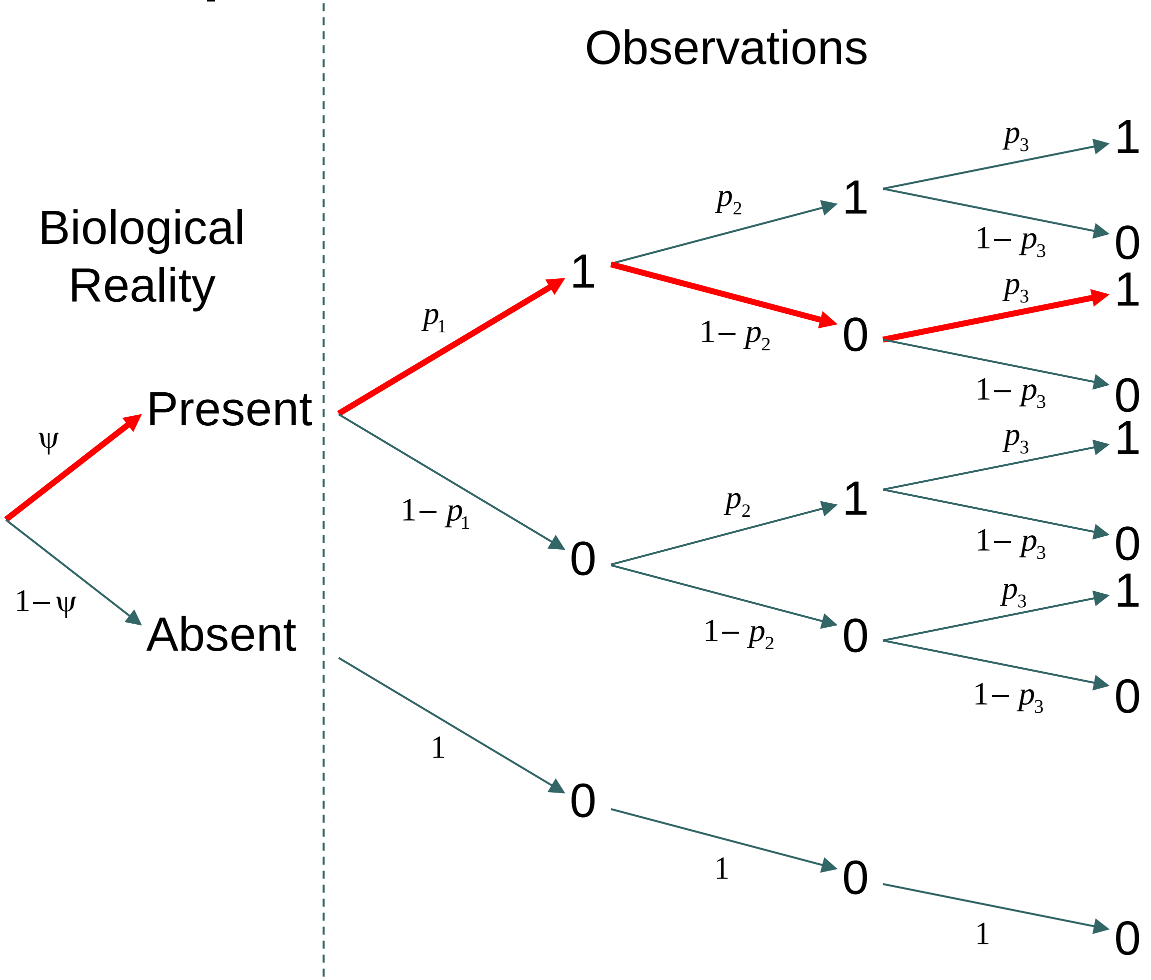
Mathematical translation:

$$\Pr\left(\mathbf{h}_2 = 000\right) = \psi \prod_{j=1}^{3} \left(1 - p_j\right) + \left(1 - \psi\right)$$

Observed Data Likelihood

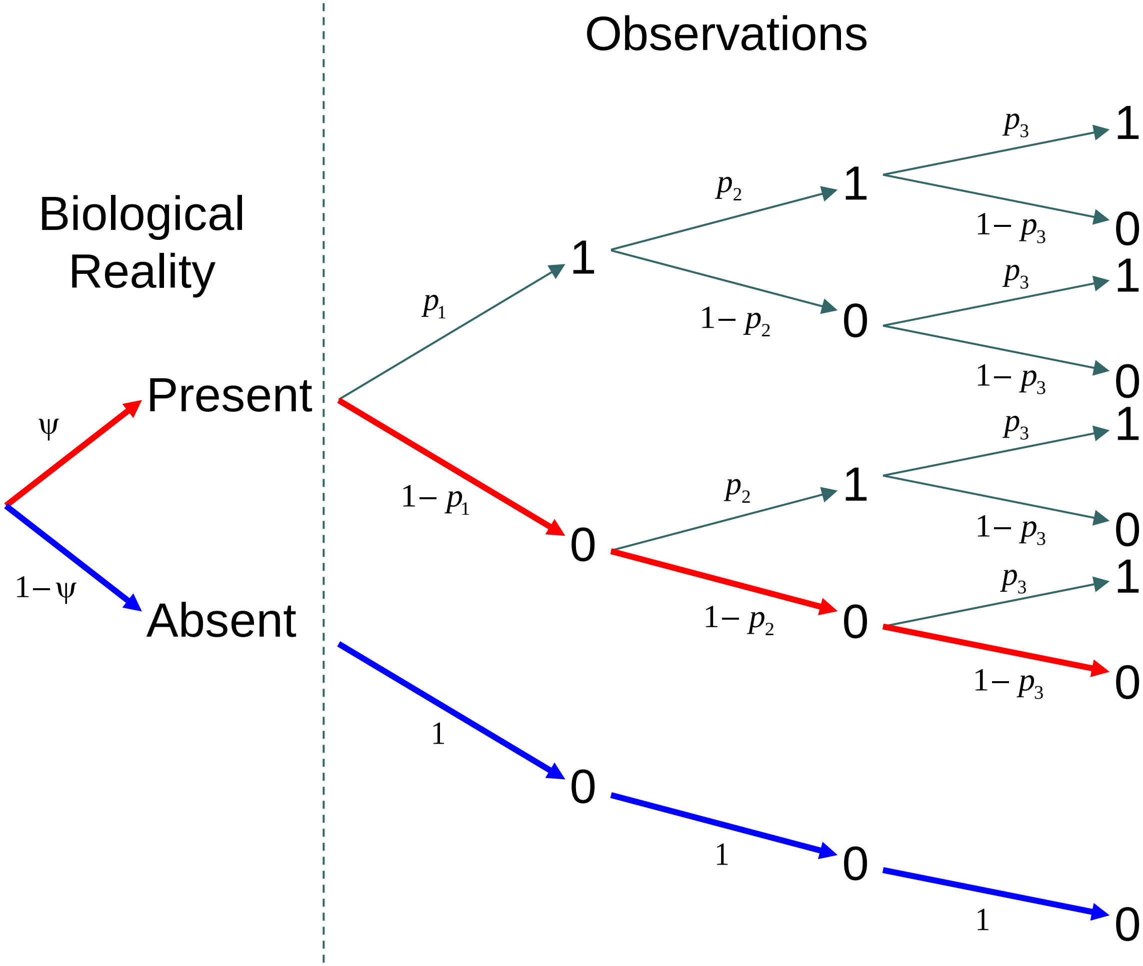


Observed Data Likelihood



101 $\psi p_1 \left(1-p_2\right) p_3$

Observed Data Likelihood



000 $\psi(1-p_1)(1-p_2)(1-p_3)$

000 $1-\psi$

Observed Data Likelihood

- Model likelihood is the product of the probability statements.

$$ODL\left(\psi,\mathbf{p}\,\middle|\,\mathbf{h}\right)=\Pr\left(\mathbf{h}\,\middle|\,\psi,\mathbf{p}\right)=\prod_{i=1}^{s}\Pr\left(\mathbf{h}_i\right)$$

- Likelihood can be maximized to obtain MLE's, or used within a Bayesian framework.

Complete Data Likelihood

- The data we wish we had!
- z_i is true presence/absence of species at unit *i*
- Bernoulli random variable with Pr(success) = ψ

Complete Data Likelihood

- $\Pr(z_i = 1) = \psi$
- $\Pr(z_i = 0) = 1 - \psi$
- $\Pr(z_i | \psi) = \psi^{z_i} (1 - \psi)^{1 - z_i}$
- Compare with: $\binom{n}{x} p^x (1 - p)^{n - x}$

Complete Data Likelihood

- h_{ij} is detection/nondetection of species (given presence) in survey *j* of unit *i*
  - Bernoulli random variable with Pr(success) = p

$$\Pr\left(h_{ij} \middle| p_j, z_i = 1\right) = p_j^{h_{ij}} \left(1 - p_j\right)^{1-h_{ij}}$$

Complete Data Likelihood

- Combining these terms, the overall likelihood can be expressed as:

$$CDL(\mathbf{p}, \psi \mid \mathbf{h}, \mathbf{z}) = \prod_{i=1}^{s} \left\{ \prod_{j=1}^{k} \left[\Pr\left(h_{ij} \mid p_j, z_i = 1 \right) \right] \Pr\left(z_i \mid \psi \right) \right\}$$

$$= \prod_{i=1}^{s} \left\{ \left[\prod_{j=1}^{k} p_j^{h_{ij}} \left(1 - p_j\right)^{1-h_{ij}} \right]^{z_i} \psi^{z_i} \left(1 - \psi\right)^{1-z_i} \right\}$$

- Note that many terms will disappear

Complete Data Likelihood

- For example, if z_i = 1 and $\mathbf{h}_i$ = 101

$$CDL\left(\mathbf{p}, \psi \middle| \mathbf{h}_i, z_i\right) = \left[\prod_{j=1}^{k} p_j^{h_{ij}} \left(1 - p_j\right)^{1-h_{ij}}\right]^{z_i} \psi^{z_i} \left(1-\psi\right)^{1-z_i}$$

$$= \psi p_1 \left(1 - p_2\right) p_3$$

Complete Data Likelihood

- For example, if z_i = 1 and $\mathbf{h}_i$ = 000

$$CDL\left(\mathbf{p},\psi \,\middle|\, \mathbf{h}_i, z_i\right) = \left[\prod_{j=1}^{k} p_j^{h_{ij}} \left(1-p_j\right)^{1-h_{ij}}\right]^{z_i} \psi^{z_i} \left(1-\psi\right)^{1-z_i}$$

$$= \psi \prod_{j=1}^{3} \left(1-p_j\right)$$

Complete Data Likelihood

- For example, if $z_i = 0$ and $\mathbf{h}_i = 000$

$$CDL\left(\mathbf{p}, \psi \middle| \mathbf{h}_i, z_i\right) = \left[\prod_{j=1}^{k} p_j^{h_{ij}} \left(1 - p_j\right)^{1-h_{ij}}\right]^{z_i} \psi^{z_i} \left(1 - \psi\right)^{1-z_i}$$

$$= \left(1 - \psi\right)$$

Complete Data Likelihood

- Unfortunately, the true values will typically be unknown
- Solutions:
  - Replace with expected values
    - Expectation – Maximization (EM) Algorithm
  - Replace with imputed values (data augmentation)
    - Markov Chain Monte Carlo (MCMC)

Complete Data Likelihood

- CDL can also be expanded to include units that were never surveyed

$$CDL\left(\mathbf{p},\psi \middle| \mathbf{h},\mathbf{z}\right)=\prod_{i=1}^{s}\left\{\left[\prod_{j=1}^{k}p_j^{h_{ij}}\left(1-p_j\right)^{1-h_{ij}}\right]^{z_i}\psi^{z_i}\left(1-\psi\right)^{1-z_i}\right\}$$

$$\times\prod_{i=s+1}^{S}\psi^{z_i}\left(1-\psi\right)^{1-z_i}$$

Covariates

- Season-specific
  - constant within a season, but may vary between seasons.
    - e.g., habitat type, patch size, generalized weather patterns

- Survey-specific
  - may vary between surveys.
    - e.g., local environmental conditions, observers

Covariates

- Occupancy and detection probabilities may be functions of season-specific covariates (via logit link, say).

$$\text{logit}\left(\psi_i\right) = \text{a} + \text{b}x_i$$

$$\text{logit}\left(p_{ij}\right) = \text{c} + \text{d}x_i$$

Covariates

- Detection probabilities may also be a function of survey-specific covariates.

$$\text{logit}\left(p_{ij}\right) = \mathrm{c} + \mathrm{d}x_i + \mathrm{e}z_{ij}$$

Covariates

- Covariates may be continuous or categorical.

- Advisable to standardize continuous covariates on to some meaningful scale such that covariates are approximately symmetrically distributed about zero.

$$x_i^* = \frac{x_i - a}{b}$$

- *z*-transformation can be done within PRESENCE.

Covariates

- Categorical covariates with *m* categories should be represented with *m* – 1 indicator (dummy) variables.
  - e.g., if 4 habitat types, use 3 indicator variables; *HabA*, *HabB*, *HabC*, with habitat D considered the ‘standard’.
- However, suggest all *m* indicator variables be included in data file.

‘Missing’ Observations

- Implicit assumption that j^{th} surveys of all units are conducted at (approximately) the same time; possibly unlikely in practice.

- Camera failure or destruction may result in some units not being surveyed during some occasions.

‘Missing’ Observations

	Day				
Unit	1	2	3	4	5
1	1	0	1	-	0
2	-	0	-	1	1

$$\Pr\left(\mathbf{h}_1 = 101-0\right) = \psi p_1 \left(1 - p_2\right) p_3 \left(1 - p_5\right)$$

$$\Pr\left(\mathbf{h}_2 = -0-11\right) = \psi \left(1 - p_2\right) p_4 p_5$$

‘Missing’ Observations

- Survey-specific covariates can only be missing if associated detection survey is also missing.
- Season-specific covariates cannot be missing.

Pr(occupied|nondetection)

- "Given the species was not detected, is the unit occupied?" is sometimes of interest.
- Can be derived from modelling results using Bayes theorem.
- Referred to as "conditional" (on nondetection) occupancy

Pr(occupied|nondetection)

$$\Pr\left(\text{occupied}\,\middle|\,\text{nondetection}\right) = \frac{\Pr\left(\text{occupied \& nondetection}\right)}{\Pr\left(\text{nondetection}\right)}$$

$$= \frac{\psi_i \prod_{j=1}^{K}\left(1-p_{ij}\right)}{\psi_i \prod_{j=1}^{K}\left(1-p_{ij}\right) + \left(1-\psi_i\right)}$$

- Standard errors can be derived with the delta method.

Single Season Model

Part II

Model Assumptions

- Closure.
- Surveys are independent.
- No unmodelled heterogeneity.
- Species identified correctly (no false detections)

What if Occupancy Changes?

- Can formally test by grouping sampling occasions within the season and using multiseason models
- But changes do not always cause problems
- If species physically occupies units at random within a season, ‘occupancy’ parameter relates to probability a unit is *used* by the species.

What if Occupancy Changes?

$$\Pr\left(\text{detect species at unit } i \text{ in survey } j\right) =$$

$$\Pr\left(\text{uses unit } i \text{ during season}\right)$$

$$\times \Pr\left(\text{physically present at unit at } j \,\middle|\, \text{uses unit}\right)$$

$$\times \Pr\left(\text{detection in survey } j \,\middle|\, \text{present and uses unit}\right)$$

- Closure implies 2^{nd} component = 1.
- When changes are random, 'detection probability' is the product of 2^{nd} and 3^{rd} components
- Recall that 'detection probability' also has spatial and camera components in camera-trap studies

What if Occupancy Changes?

- Immigration/emigration only; ‘occupancy’ parameter relates to probability is present at a unit at end/beginning of season respectively.
  - should allow detection probability to vary within a season.
  - should pool some survey occasions.

- Open models permitting staggered entry and departure times (only 1 entry/depart per season)
  - Single-season models (Kendall et al. 2013)
  - Multiple seasons (Chambert et al. 2015)

What if Occupancy Changes?

- Some non-random changes may cause biases.
- Is a ‘season’ defined appropriately?
- Is time between surveys appropriate?
- Area of active research.

Lack of Independence

- Surveys are not independent if the outcome of survey A is dependent upon the outcome of survey B.
- Some forms of dependence may be accommodated with good designs or modelling.
- In some instances, parameter estimates may be OK, but standard errors too small.

Lack of Independence

- To account for ‘trap response’ (unusual in CT studies); define a survey-specific covariate that equals 1 for all surveys after first detection at a site, 0 otherwise.

	X_{ij}			
h	1	2	3	4
0101	0	0	1	1
1101	0	1	1	1
0001	0	0	0	0
0000	0	0	0	0

Accounting for Lack of Closure/Independence

- “Spatial Correlation” option in PRESENCE (Hines et al. 2010, 2014)

- Motivating example: surveying tigers on trails in India

- Introduce 2 new parameters
  - θ – Pr(tiger on trail given not on trail in previous segment): *thta0*
  - θ' – Pr(tiger on trail given on trail in previous segment): *thta1*
  - θ'' – Pr(tiger on trail in first segment)

Accounting for Lack of Closure/Independence

$$\Pr\left(\mathbf{h}_i = 01110\right) = \psi \begin{bmatrix} \theta''\left(1 - p_1\right)\theta' p_2 \theta' p_3 \theta' p_4 \theta'\left(1 - p_5\right) \\ +\theta''\left(1 - p_1\right)\theta' p_2 \theta' p_3 \theta' p_4 \left(1 - \theta'\right) \\ +\left(1 - \theta''\right)\theta p_2 \theta' p_3 \theta' p_4 \theta'\left(1 - p_5\right) \\ +\left(1 - \theta''\right)\theta p_2 \theta' p_3 \theta' p_4 \left(1 - \theta'\right) \end{bmatrix}$$

Accounting for Lack of Closure/Independence

$$\Pr\left(\mathbf{h}_i = 01010\right) = \psi \left[\begin{array}{l} \theta''\left(1-p_1\right)\theta' p_2 \theta'\left(1-p_3\right)\theta' p_4 \theta'\left(1-p_5\right) \\ +\theta''\left(1-p_1\right)\theta' p_2 \theta'\left(1-p_3\right)\theta' p_4\left(1-\theta'\right) \\ +\theta''\left(1-p_1\right)\theta' p_2\left(1-\theta'\right)\theta p_4 \theta'\left(1-p_5\right) \\ +\theta''\left(1-p_1\right)\theta' p_2\left(1-\theta'\right)\theta p_4\left(1-\theta'\right) \\ +\cdots \end{array} \right]$$

Accounting for Lack of Closure/Independence

- Could also possibly be applied to account for temporal correlation or non-random changes in occupancy within a season.

- Example: useful for modeling N.A. Breeding Bird Survey data

Unmodelled Heterogeneity

In occupancy probabilities

- parameter estimates should still be valid as average values across the units surveyed.

In detection probabilities

- occupancy will be underestimated.
- covariates may account for some sources of variation.

Incorporating Heterogeneity

- Finite Mixtures
  - Assume occupied units consist of G groups.
  - Each group has a different p.
  - Group membership is unknown hence a unit may belong to any of the G groups.

$$\Pr\left(\mathbf{h}_1 = 10\right) = \psi\left[\pi_1 p_1\left(1 - p_1\right) + \left(1 - \pi_1\right) p_2\left(1 - p_2\right)\right]$$

Incorporating Heterogeneity

- Random Effects
  - Assume p_i is a random value from a continuous probability distribution (e.g., beta distribution, logit-normal).
  - Closed form expressions possible for some distributions.
  - Easily implemented using WinBUGS.

Abundance Induced Heterogeneity

- Differences in the local abundance of the species between units may induce heterogeneity in detection probability.

- Royle and Nichols (2003) suggested an extension of the above method to accommodate this.

$$p_{ij}^{[N]} = 1 - \left(1 - r_j\right)^{N_i}$$

Abundance Induced Heterogeneity

- Local abundance is unknown, but a spatial distribution could be assumed (e.g., Poisson).

$$\Pr\left(\mathbf{h}_1 = 101\right) = Po\left(1;\mu\right) p_{i1}^{[1]} \left(1 - p_{i2}^{[1]}\right) p_{i3}^{[1]}$$

$$+ Po\left(2;\mu\right) p_{i1}^{[2]} \left(1 - p_{i2}^{[2]}\right) p_{i3}^{[2]}$$

$$+ \ldots$$

$$= \sum_{l=1}^{\infty} Po\left(l;\mu\right) p_{i1}^{[l]} \left(1 - p_{i2}^{[l]}\right) p_{i3}^{[l]}$$

Abundance Induced Heterogeneity

$$\Pr\left(\mathbf{h}_2 = 000\right) = Po\left(1;\mu\right)\prod_{j=1}^{3}\left(1 - p_{ij}^{[1]}\right)$$

$$+Po\left(2;\mu\right)\prod_{j=1}^{3}\left(1 - p_{ij}^{[2]}\right)$$

$$+\ldots$$

$$+Po\left(0;\mu\right)$$

$$= \sum_{l=1}^{\infty} Po\left(l;\mu\right)\prod_{j=1}^{3}\left(1 - p_{ij}^{[l]}\right) + Po\left(0;\mu\right)$$

Abundance Induced Heterogeneity

- Occupancy is now a derived parameter: $(1-e^{-\mu})$.

- Implicit assumption that the number of animals at a unit is constant.

- Care must be taken in how 'abundance' should be interpreted.

Abundance Induced Heterogeneity

- Similar model can result through random effect on complementary log-log link function for p.

- A good approach for incorporating heterogeneity in detection for occupancy estimation, less reliable if inferences about abundance are desired.

Species Misidentification (False Positives)

- If species is falsely detected then occupancy could be overestimated.
- Much recent work; now a separate lecture
- Perhaps not so important with camera-trap data?

Assessing Model Fit

- MacKenzie and Bailey (2004) suggest a test based on the observed and expected number of sites with each possible detection history.
- The expected number is predicted by the model, which may include covariates.

Assessing Model Fit

- $E_h = \sum_{i=1}^{s} \Pr(\mathbf{h}_i)$

- For example, for the history 101:

Unit	$\hat{\psi}_i$	$\hat{p}_i$	$\Pr(101)$
1	0.30	0.76	0.042
2	0.35	0.74	0.050
3	0.40	0.72	0.058
4	0.45	0.70	0.066
Total			0.216

Assessing Model Fit

- $$TS = \sum_h \frac{\left(O_h - E_h\right)^2}{E_h}$$
- Use parametric bootstrap to assess the evidence for lack of fit.
- $$\hat{c} = \frac{TS}{\overline{TS}_B}$$
  - used to adjust SE's and AIC values.

Assessing Model Fit

- Test has been shown to perform well to identify poor model structure w.r.t. detection probabilities, but not occupancy probabilities.
- Found to have generally low-power, especially for the sample sizes expected in many applications (s<100).
- Recommend ≥10,000 bootstraps.

Assessing Model Fit

- Test should be conducted on the most complicated model under consideration (the global model) and results applied to all models in candidate set.
  - e.g., if from global model $\hat{c} = 1.53$, this value is used to adjust AIC values and SE's for all models.

Assessing Model Fit

- Posterior predictive checks could be used within a Bayesian setting to determine whether observed features of the data is “similar” to what a fitting model would expect.

Finite Population

- In some circumstances, the sample of s units may constitute a large fraction of the population of interest.
- Strictly speaking, the methods above estimate the *probability* of occupancy.
  - an underlying characteristic of the population.
- The *proportion* of units occupied is a realisation of this process.

Finite Population

- Distinction between them is of little practical consequence for ‘infinite’ populations, but may be for ‘finite’ populations.

- SE’s will be too large if not accounted for.

Finite Population

- The proportion of occupied sites could be calculated as:

$$\left(s_D + \sum_{i=s_D+1}^{s} \hat{\psi}_i^c + \sum_{i=s+1}^{S} \hat{\psi}_i \right) \Big/ S$$

- SE derived from delta method

Finite Population

- When no covariates in the model, variance (ie SE^2) of the proportion is:

$$\frac{\left\{\begin{array}{c}\left(s-s_D\right)\left[\hat{\psi}^c\left(1-\hat{\psi}^c\right)+\left(s-s_D-1\right)Var\left(\hat{\psi}^c\right)\right]+\\ \left(S-s\right)\left[\hat{\psi}\left(1-\hat{\psi}\right)+\left(S-s-1\right)Var\left(\hat{\psi}\right)\right]+\\ \left(s-s_D\right)\left(S-s\right)Cov\left(\hat{\psi}^c,\hat{\psi}\right)\end{array}\right\}}{S^2}$$

Finite Population

- Alternatively, easily implemented using the data augmentation approach.

- Presence/absence of the species is predicted for units where species not detected.

- Occupancy state can also be predicted for units that were never surveyed.

Finite Population

```
model {
    for (i in 1:s) {
        z[i] ~ dbern(psi)
        z1[i] <- z[i]+1
        for (j in 1:k) {
            h[i,j] ~ dbern(p[j,z1[i]])
        }
    }
    psi.fs<-sum(z[])/s

    ## define prior distributions for model parameters
    psi~dunif(0,1)
    for (j in 1:k) {
        p[j,1] <- 0
        p[j,2] ~ dunif(0,1)
    }
}
```

Post. Distn. ψ

mean = 0.56, sd = 0.10

{2.5, 50, 97.5} %iles =

{0.37, 0.56, 0.78}

Post. Distn. ψ*

mean = 0.56, sd = 0.08

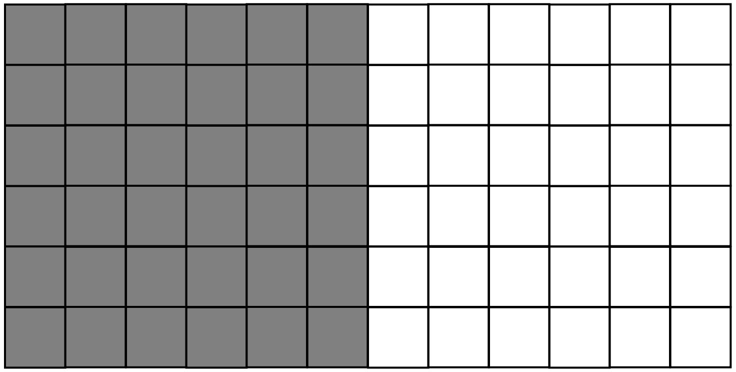
{2.5, 50, 97.5} %iles =

{0.46, 0.54, 0.74}

Spatial Correlation

- Probability of occupancy at a unit may depend upon whether a ‘neighbouring’ unit is also occupied.
- Some forms of clustering may be well explained by covariates.
- May not be important to account for if interest is in an overall measure of occupancy.
- May be more important if interest is in maps or predictions of unit-level occupancy.

Spatial Correlation



- What fraction of cells are occupied?
- What would be the appropriate SE?

Spatial Correlation

- Recent simulations have confirmed that overall estimates are unbiased, with appropriate standard errors, without accounting for spatial correlation when units are sampled randomly.

Spatial Correlation

- Sargeant et al. (2005) suggested an approach using image restoration methods.
  - doesn’t generalize easily to incorporate covariates.

- Magoun et al. (2007) assumed spatially correlated residuals with logistic regression.

Spatial Correlation

- Alternatively, could model occupancy with the autologistic function.
  - essentially the logit link with an 'effect' related to the number of occupied neighboring units.

$$\text{logit}(\psi_i) = \beta_0 + \beta_1 x_{1i} + \beta_2 g_i$$

    - g_i is a function of the occupied 'neighboring' units

- But the exact occupancy state of units will often be unknown…

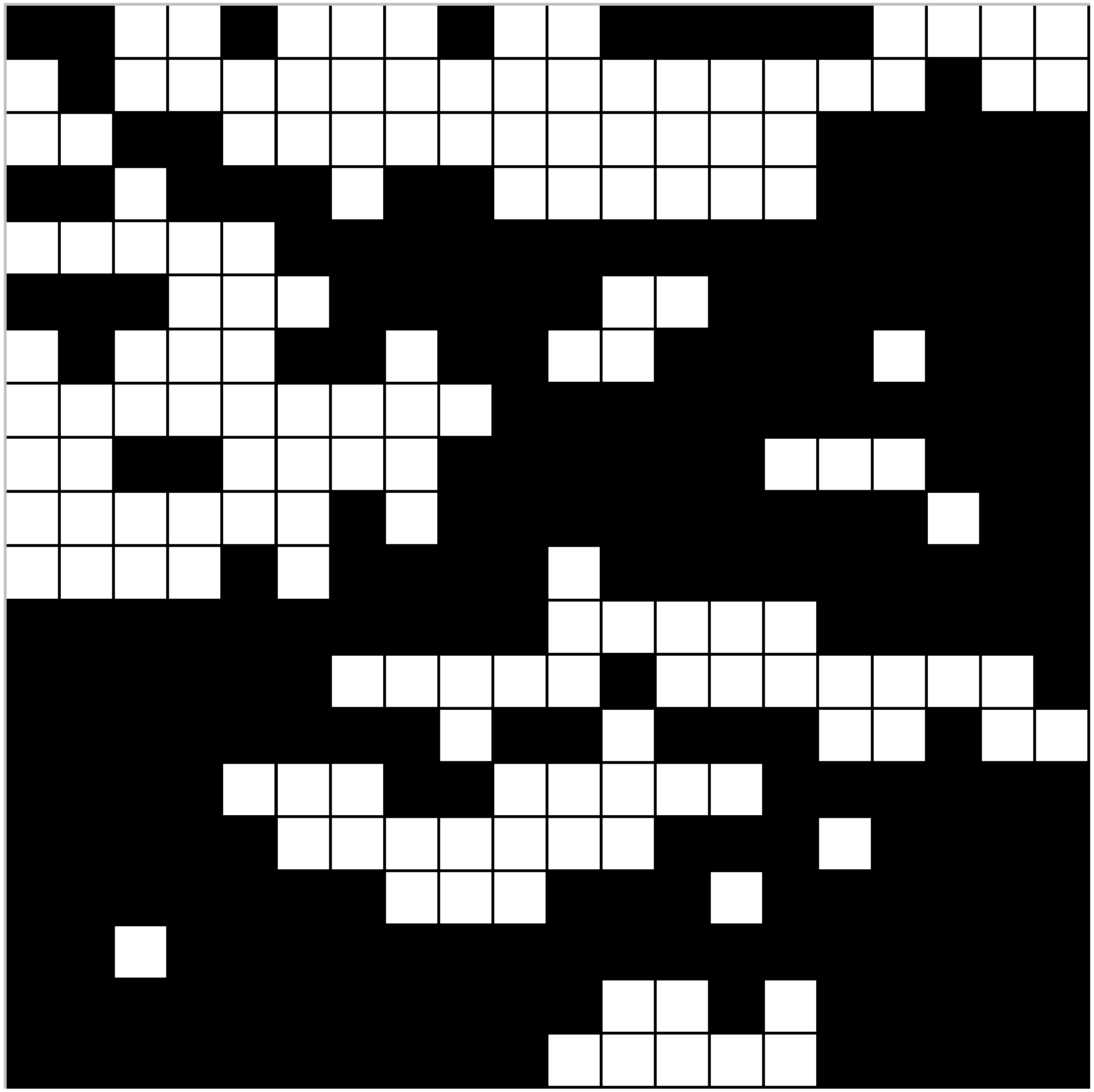
Spatial Correlation

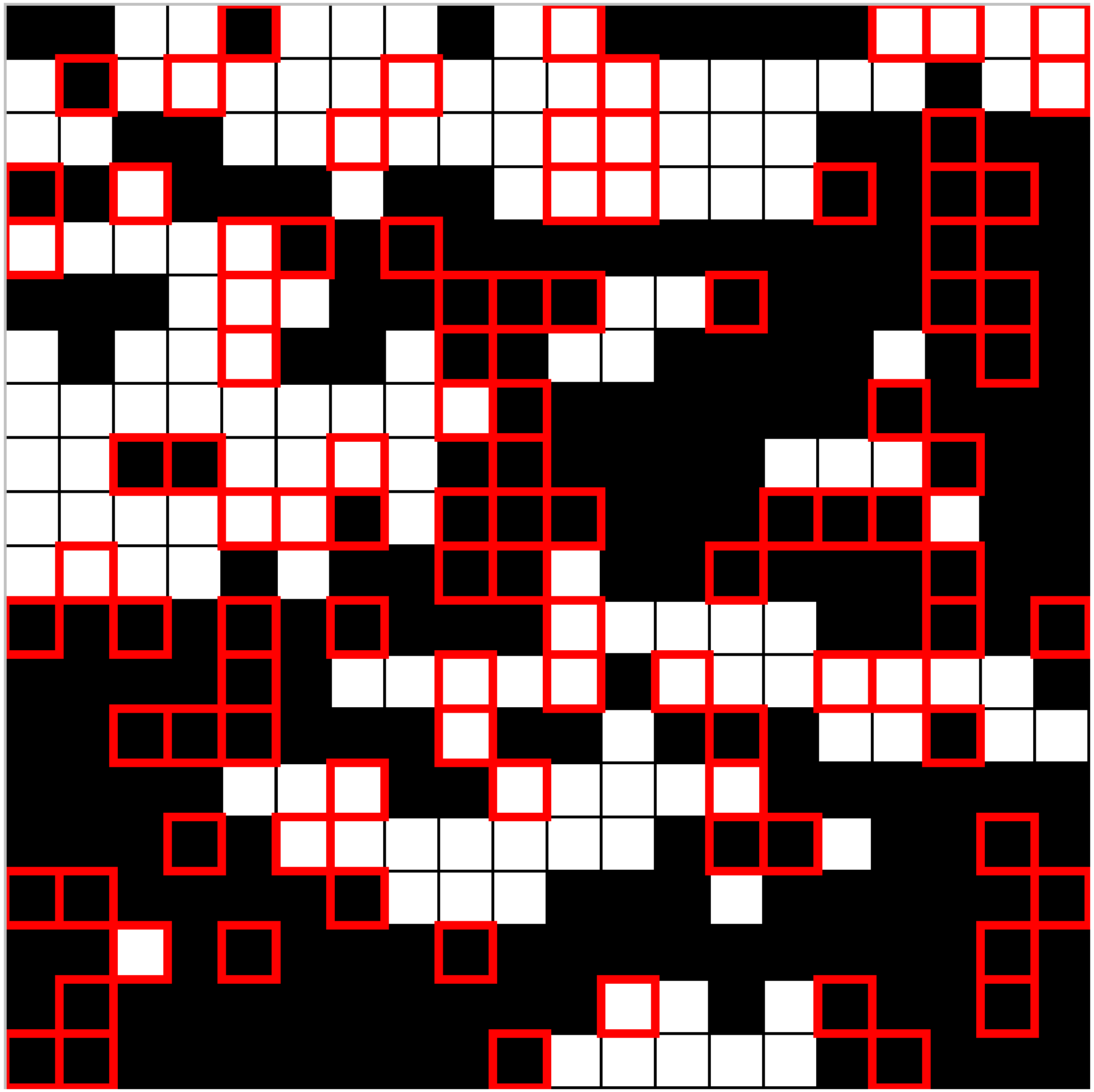
- Theoretically, could integrate over all unknown occupancy states for all units in the landscape (i.e., develop ODL).

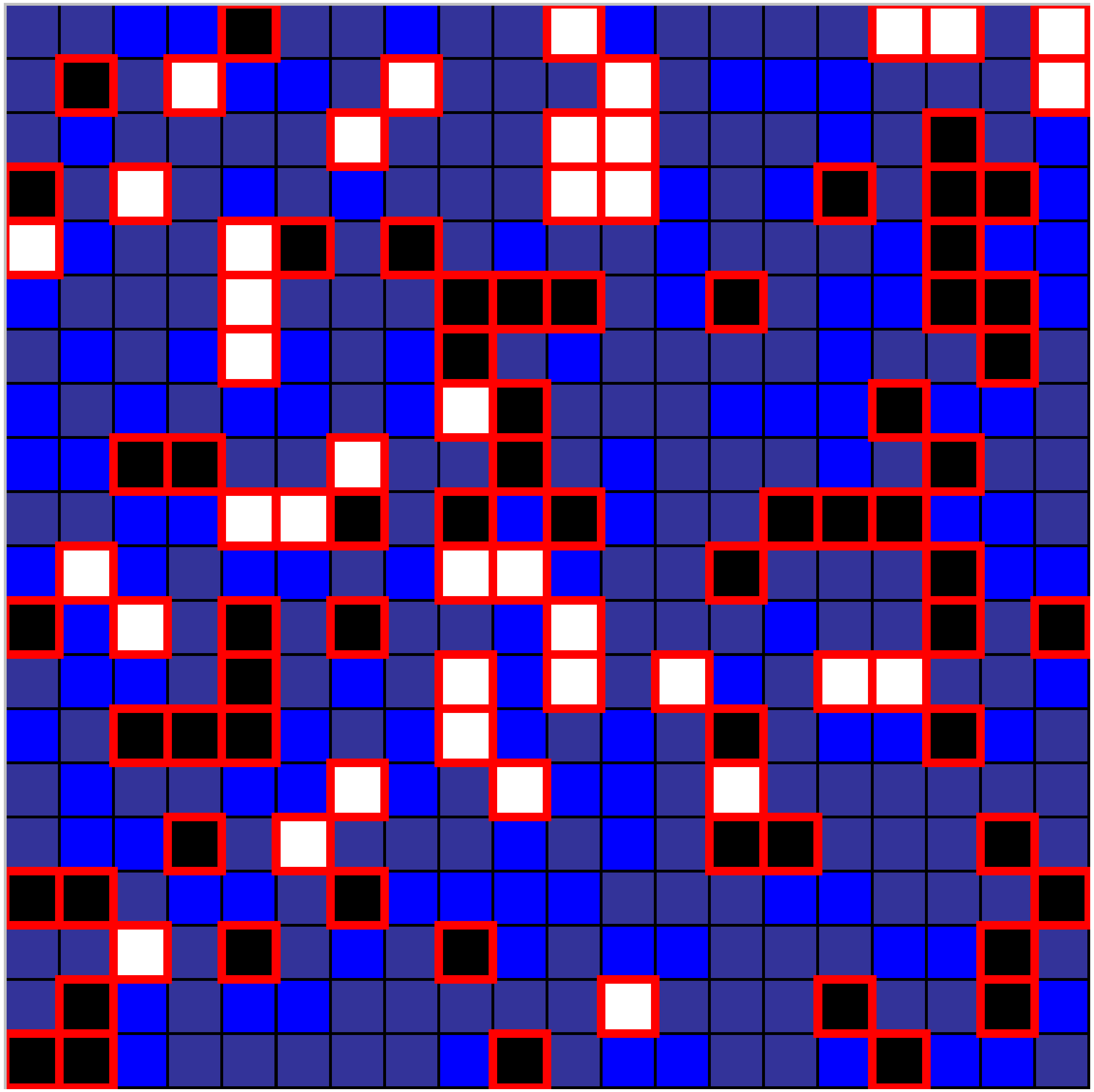
- Practically, much easier to develop the complete data likelihood and use data augmentation or EM algorithm.

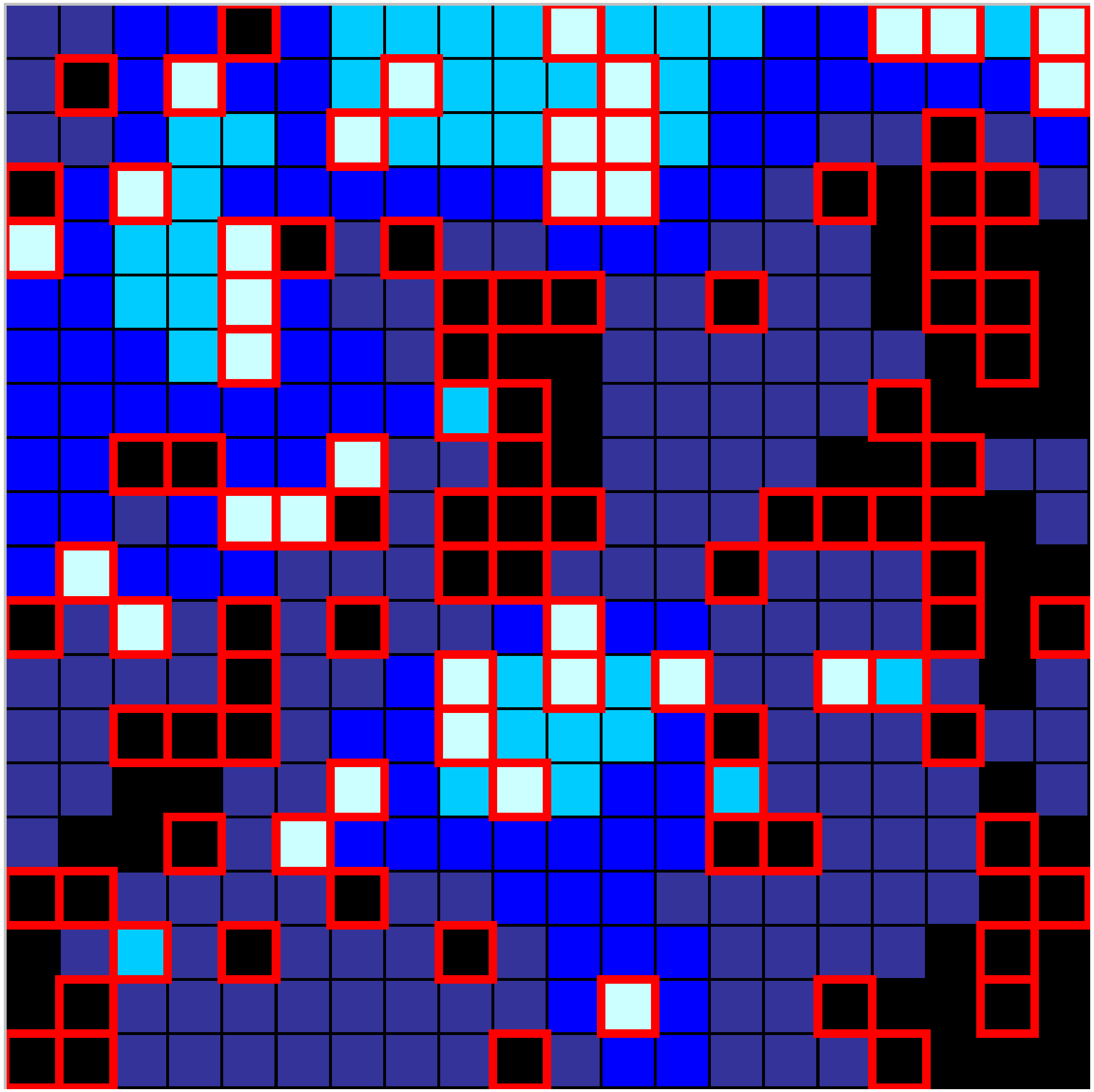
Spatial Correlation

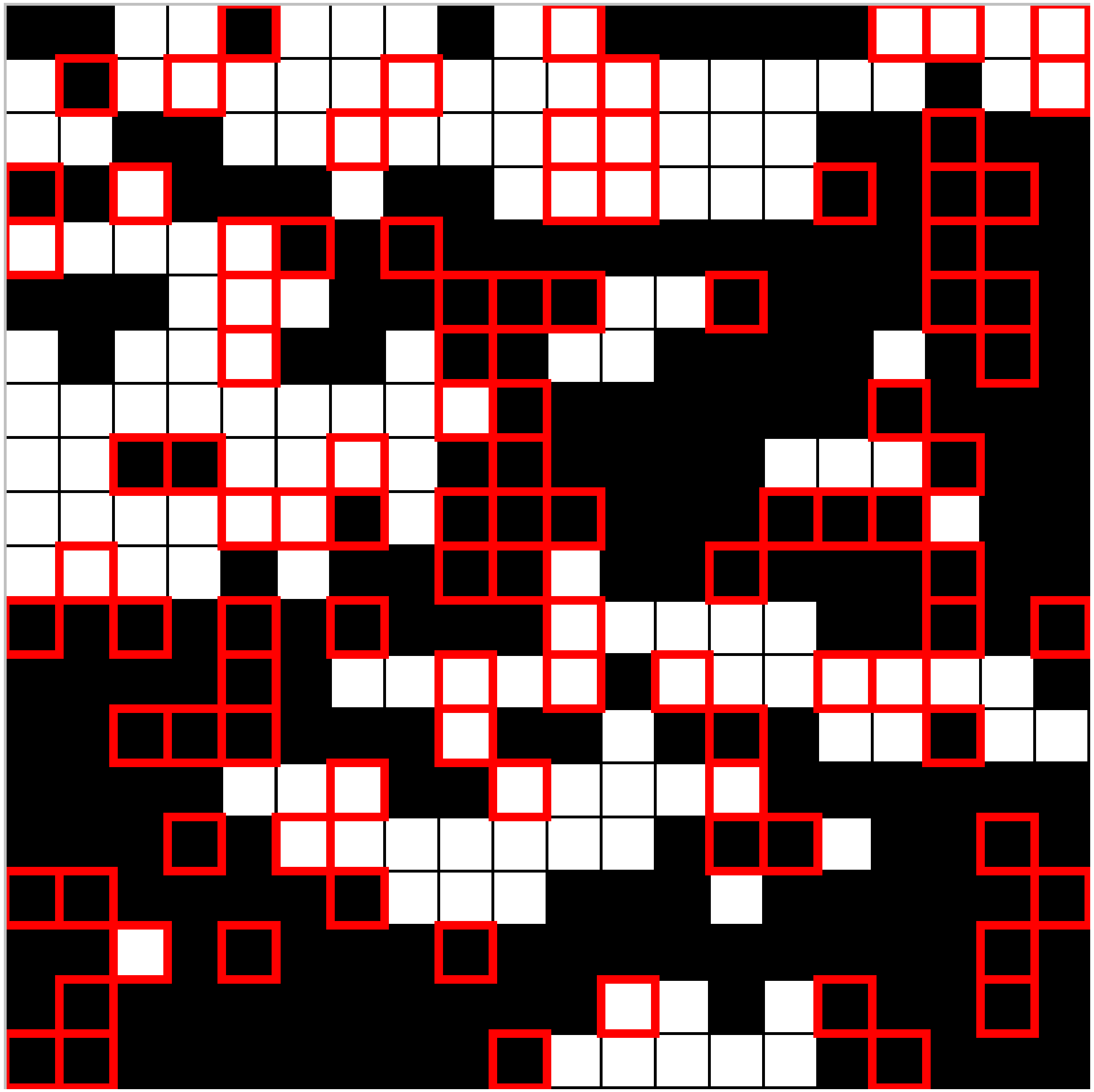
- Detection probability may also be a function of the number of neighboring occupied units (e.g., harder to detect a species at the edge of it's range).











Summary

- Historically, investigating *patterns* in occupancy has been the main focus of such studies.
- A suite of flexible methods is now available that account for:
  - detectability
  - covariates
  - unequal sampling effort
  - heterogeneity
  - finite populations
  - spatial correlation
- Useful for assessing a snapshot of a population, but not for understanding the underlying dynamics.